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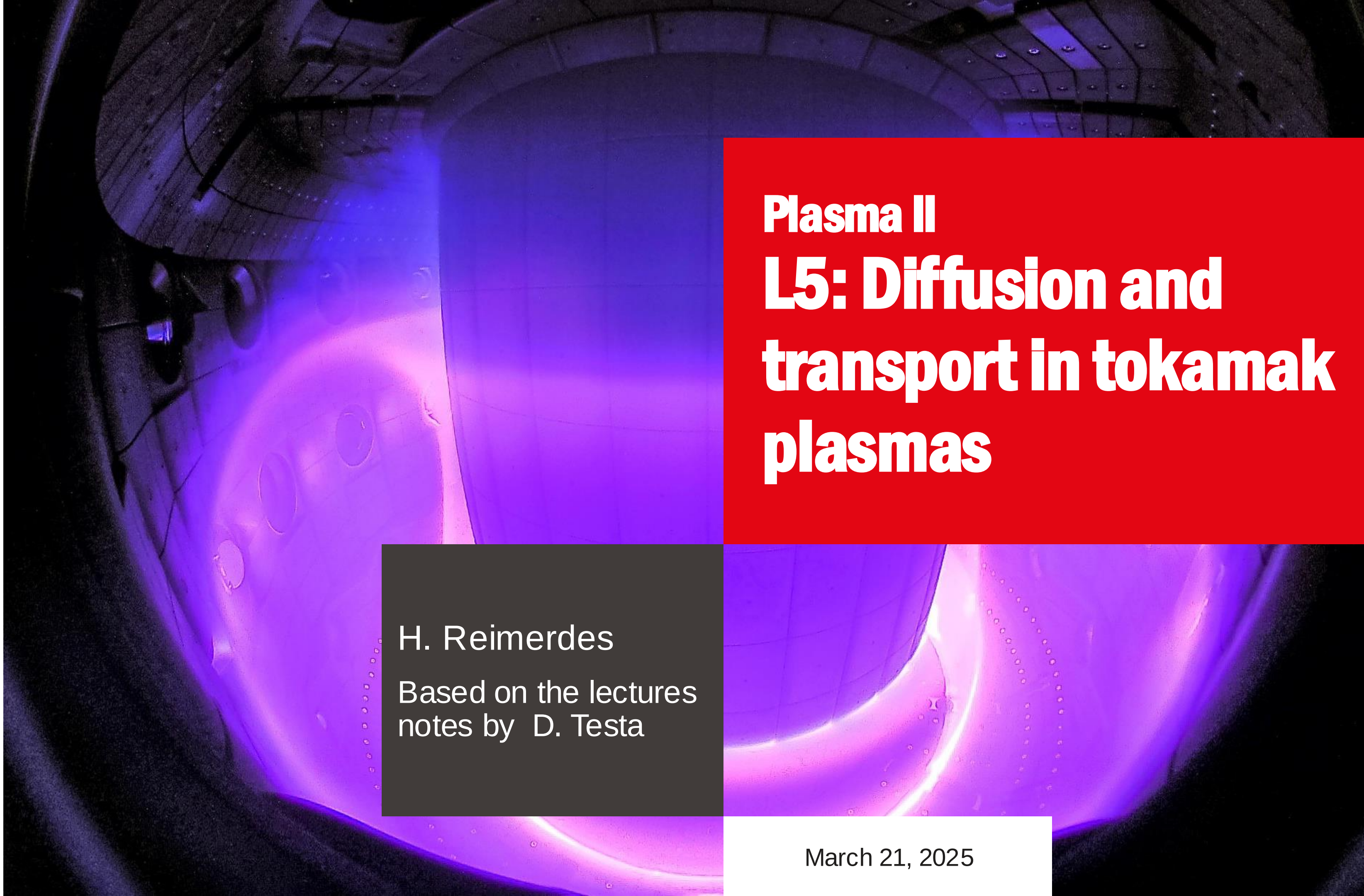

# Plasma II
# L5: Diffusion and transport in tokamak plasmas


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Based on the lectures notes by D. Testa


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# Outline



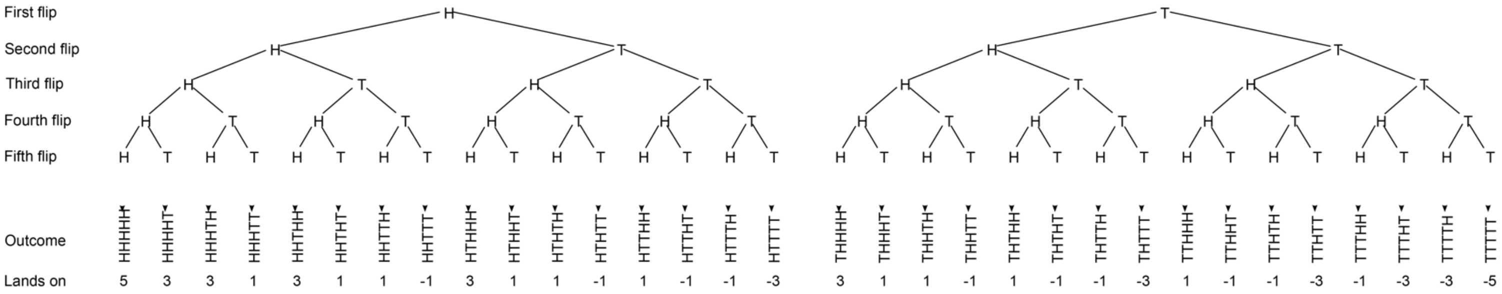
1. General principle of diffusive transport: random walk
2. Charge-neutrality ➔ ambipolar diffusion
3. Classical diffusion and transport in a tokamak (highly ionised, magnetised plasma)
4. Neo-classical diffusion and transport in a tokamak (particle description)
5. Turbulent transport in tokamaks ➔ anomalous diffusion

## Material

- See also EPFL MOOC “Plasma physics: Applications” #7d,e
  - https://learning.edx.org/course/course-v1:EPFLx+PlasmaApplicationX+1T_2018/home
- Wesson, *Tokamaks* - Third Edition, Ch. 2.4-2.6, 3.9-3.12, 4.1-4.8, 4.12-4.14, 4.17-4.18, 8.1-8.2

# Classical diffusion: random walk determined by a coin flip

- Example of 1D random walk diffusion: Random walk, which starts at 0 and at each step moves +1 (‘heads’) or −1 (‘tails’) with equal probability

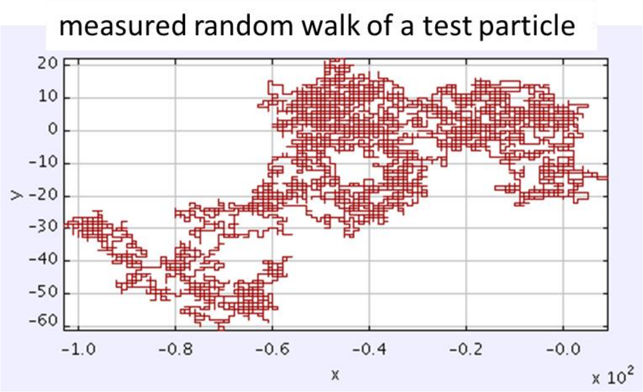


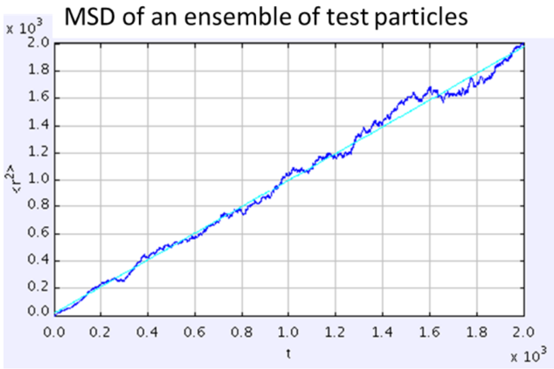

- Five flips: marker could now be on $\{\pm 1, \pm 3, \pm 5\}$
- Overall probability: 10×(+1), 10×(-1), 5×(+3), 5×(-3), 1×(+5), 1×(-5) out of 32
  - Average displacement: $\langle \Delta x \rangle = 0$ (!)
  - Root mean square (RMS) displacement: $\sqrt{\langle (\Delta x)^2 \rangle} = \sqrt{5}$
- **General: *N* steps with step size $\delta_{\text{step}}$ leads to $\sqrt{\langle (\Delta x)^2 \rangle} = \delta_{\text{step}} \sqrt{N}$**

# Classical diffusion: random walk across cellular membranes




- Example of 2D random walk diffusion




- Mean squared displacement $\langle(\Delta x)^2\rangle$ for an ensemble of test particles is proportional to time $\Delta t$ → i.e. number of steps $N$
  - With time between steps $\tau_{\text{step}}$, time for $N$ steps is $\Delta t = N\tau_{\text{step}}$

$$\sqrt{\langle(\Delta x)^2\rangle} = \delta_{\text{step}}\sqrt{N} \quad \Rightarrow \quad \langle(\Delta x)^2\rangle = \delta_{\text{step}}^2\, N = \frac{\delta_{\text{step}}^2}{\tau_{\text{step}}}\Delta t$$

Diffusion coefficient $D$

# From displacement to transport

- Diffusion coefficient: $D = \frac{\delta_{\text{step}}^2}{\tau_{\text{step}}} = \nu_{\text{step}} \delta_{\text{step}}^2$
  - Step frequency $\nu_{\text{step}} = \tau_{\text{step}}^{-1}$
  - (Average) step size $\delta_{\text{step}}$
- Displacement becomes only transport when there is a gradient

$$\bar{\Gamma} = -D \nabla n$$

*Fick's first law (of diffusion)*

- Random walk analogy can be equally applied to heat conduction

$$\bar{q} = -\underbrace{n k_B \chi}_{} \nabla T$$

Thermal conductivity *k* [W/(m·K)]

# Diffusive transport 'closes' the continuity equation



- Continuity equation (see L2)

$$\frac{\partial n}{\partial t} + \nabla(n \cdot \bar{V}) = S$$

- Assume diffusive transport

$$\frac{\partial n}{\partial t} - \nabla(D \nabla n) = S$$

*Diffusion equation*

- Estimate of confinement time: $\tau_{\text{conf}} \approx a^2/D$
  - Characteristic system dimension *a*

# Diffusion in gases

- Movement of gas atoms *A* from a region of a high concentration to a region of low concentration
- Only force on particles arises from collisions
  - Step frequency $\nu$ ➔ Collision frequency $\nu_C$
    - Collisions need to exchange momentum with background gas for a diffusive transport of particle species *A* (!)
  - Step-size δ ➔ Mean free path $\lambda_{\mathrm{MFP}} = v_{\mathrm{th}}\,\tau_{\mathrm{C}} = v_{\mathrm{th}}/\nu_{\mathrm{C}}$
- Diffusion coefficient

$$D_{\mathrm{GAS}} = \nu_{\mathrm{C}}\lambda_{\mathrm{MFP}}^2 = \nu_{\mathrm{C}}\left(\frac{v_{\mathrm{th}}}{\nu_{\mathrm{C}}}\right)^2 = \frac{v_{\mathrm{th}}^2}{\nu_{\mathrm{C}}} = \frac{k_{\mathrm{B}}T}{m\,\nu_{\mathrm{C}}}$$

➢ **Collisions slow down diffusive transport in a gas**

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1. General principle of diffusive transport: random walk
2. **Charge-neutrality ➔ ambipolar diffusion**
3. Classical diffusion and transport in a tokamak (fully ionized, magnetized plasma)
4. Neo-classical diffusion and transport in a tokamak (particle description)
5. Turbulent transport in tokamaks ➔ anomalous diffusion

# Diffusion in a weakly ionised plasma

- Diffusion coefficient: $D = \nu_{\mathrm{step}}\delta_{\mathrm{step}}^2$
  - Step frequency $\nu_{\mathrm{step}}$ ➔ Collision frequency $\nu_C$
  - (Average) step size $\delta_{\mathrm{step}}$ ➔ Mean free path $\lambda_{\mathrm{MFP}} = v_{\mathrm{th}}/\nu_{\mathrm{C}}$
- In a *weakly ionised plasma* collisions with neutrals dominate $\nu_{\mathrm{C}} = \nu^{\mathrm{e,i/n}}$
  - Mean free paths $\lambda_{\mathrm{MFP}}$ for electrons and ions are the same

| Mean free path |
|---|
| |

➢ Collision frequencies of e and i, $\nu^{\mathrm{e,i/n}} = v_{\mathrm{th,e,i}}/\lambda_{\mathrm{MFP}}$, differ

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# Diffusion in a weakly ionised plasma

- Diffusion coefficient: $D = \nu_{\text{step}}\delta_{\text{step}}^2$
  - Step frequency $\nu_{\text{step}}$ ➔ Collision frequency $\nu_C$
  - (Average) step size $\delta_{\text{step}}$ ➔ Mean free path $\lambda_{\text{MFP}} = v_{\text{th}}/\nu_{\text{C}}$
- In a *weakly ionised plasma* collisions with neutrals dominate $\nu_{\text{C}} = \nu^{\text{e,i/n}}$
  - Mean free paths $\lambda_{\text{MFP}}$ for electrons and ions are the same ➔ $\nu^{\text{e,i/n}} = \frac{v_{\text{th,e,i}}}{\lambda_{\text{MFP}}}$

$$D_{\text{e}} = \lambda_{\text{MFP}}\, v_{\text{th,e}} \quad \text{and} \quad D_{\text{i}} = \lambda_{\text{MFP}}\, v_{\text{th,i}}$$

- For $T_e = T_{\text{i}}$,

$$\boxed{\frac{D_{\text{e}}}{D_{\text{i}}} = \frac{v_{\text{th,e}}}{v_{\text{th,i}}} = \sqrt{\frac{m_{\text{i}}}{m_{\text{e}}}} \gg 1}$$

➢ **In weakly ionised plasmas electron diffusivity is greater than ion diffusivity**

# Ambipolar diffusion and electric field




- Consider particle motion in the absence of (or parallel to) the magnetic field in the presence of a density gradient:

$$\bar{\Gamma}_i = -D_i \nabla n_i + n_i \mu_i \bar{E} \quad \text{and} \quad \bar{\Gamma}_e = -D_e \nabla n_e + n_e \mu_e \bar{E}$$

- Non-ambipolar transport leads to charge separation and the creation of an electric field that balances the transport so that $Z\bar{\Gamma}_i = \bar{\Gamma}_e = \bar{\Gamma}$

$$\bar{\Gamma} = -D_{AP} \nabla n \quad \text{with} \quad D_{AP} = \frac{\mu_i D_e - \mu_e D_i}{\mu_i - \mu_e}$$

- Using *Einstein's relation* $D_s/\mu_s = k_B T_s/q_s$ and $T_e = T_\mathrm{i}$

$$D_{AP} = \frac{(Z+1) D_i D_e}{Z D_i + D_e}$$

- With $D_e \gg Z\, D_\mathrm{i}$

$$D_{AP} \approx (Z+1) D_i \quad \text{and} \quad \bar{E}_{AP} = \frac{D_i - D_e}{\mu_i - \mu_e} \frac{\nabla n}{n} \approx \frac{k_B T}{e} \frac{\nabla n}{n}$$

Ambipolar electric field

# Ambipolar diffusion and electric field

$$D_{AP} \approx (Z+1)D_i \qquad \text{and} \qquad \bar{E}_{AP} = \frac{D_i - D_e}{\mu_i - \mu_e}\frac{\nabla n}{n} \approx \frac{k_B T}{e}\frac{\nabla n}{n}$$

- Diffusion coefficients: $D_\mathrm{i} < D_\mathrm{AP} < D_\mathrm{e}$ ➔ $\bar{E}_\mathrm{AP}$ slows down the electrons and increases ion diffusion by a factor $1+Z_i \geq 2$

# Outline

1. General principle of diffusive transport: random walk
2. Charge-neutrality → ambipolar diffusion
3. **Classical diffusion and transport in a tokamak (highly ionised, magnetised plasma)**
4. Neo-classical diffusion and transport in a tokamak (particle description)
5. Turbulent transport in tokamaks → anomalous diffusion

# Reminder: *Coulomb collisions* in a highly ionised thermal plasma (Plasma I)

| Momentum transfer | | Energy transfer | |
|---|---|---|---|
| • Electron-ion collisions $\nu_p^{e/i} = \frac{1}{3}\sqrt{\frac{2}{\pi}} n_i \frac{Z^2 e^4 \ln\Lambda}{4\pi\varepsilon_0^2 m_e^{1/2}} \frac{1}{(k_B T_e)^{3/2}}$ | ~O(1) | $\nu_E^{e/i} = 2\frac{m_e}{m_i}\nu_p^{e/i}$ | $\sim\frac{1}{1836}$ |
| • Electron-electron collisions $\nu_p^{e/e} = \frac{1}{\sqrt{2}}\nu_p^{e/i}$ | ~O(1) | $\nu_E^{e/e} = \nu_p^{e/e}$ | ~1 |
| • Ion-ion collisions $\nu_p^{i/i} = \frac{1}{\sqrt{2}}\sqrt{\frac{m_e}{m_i}}\left(\frac{T_e}{T_i}\right)^{3/2}\nu_p^{e/i}$ | $\sim\frac{1}{43}$ | $\nu_E^{i/i} = \nu_p^{i/i}$ | $\sim\frac{1}{43}$ |
| • Ion-electron collisions $\nu_p^{i/e} = \frac{n_e}{n_i}\frac{m_e}{m_i}\nu_p^{e/i}$ | $\sim\frac{1}{1836}$ | $\nu_E^{i/e} = 2\nu_p^{i/e} \cong \nu_E^{e/i}$ | $\sim\frac{1}{1836}$ |

# Classical picture of diffusion along the magnetic field* in a highly ionised plasma



- Step-size
  - Ions: $\lambda_{\mathrm{MFP,i}} = v_{\mathrm{th,i}}/\nu^{\mathrm{i}}$
  - Electrons: $\lambda_{\mathrm{MFP,e}} = v_{\mathrm{th,e}}/\nu^{\mathrm{e}}$

$$D_{||} = \frac{v_{\mathrm{th}}^2}{\nu}$$ decreases with collisionality!

- Collision frequencies

| | Momentum | Energy |
|---|---|---|
| e-i $[\nu_{\mathrm{p}}^{\mathrm{e/i}}]$ | $1$ | $\sim m_{\mathrm{e}}/m_{\mathrm{i}}$ |
| e-e $[\nu_{\mathrm{p}}^{\mathrm{e/i}}]$ | $\sim 1$ | $\sim 1$ |
| i-i $[\nu_{\mathrm{p}}^{\mathrm{e/i}}]$ | $\sim\sqrt{m_{\mathrm{e}}/m_{\mathrm{i}}}$ | $\sim\sqrt{m_{\mathrm{e}}/m_{\mathrm{i}}}$ |
| i-e $[\nu_{\mathrm{p}}^{\mathrm{e/i}}]$ | $\sim m_{\mathrm{e}}/m_{\mathrm{i}}$ | $\sim m_{\mathrm{e}}/m_{\mathrm{i}}$ |

- Collisions among particles of same species contribute to heat, but not particle transport

Electrons: $D_{||}^{\mathrm{e}} \approx v_{\mathrm{th,e}}^2/\nu_{\mathrm{p}}^{\mathrm{e/i}}$ $\quad \chi_{||}^{\mathrm{e}} \approx v_{\mathrm{th,e}}^2/\left(\nu_{\mathrm{E}}^{\mathrm{e/e}} + \cancel{\nu_{\mathrm{E}}^{\mathrm{e/i}}}\right)$ $< \nu_{\mathrm{E}}^{\mathrm{e/e}}$

Ions: $D_{||}^{\mathrm{i}} \approx v_{\mathrm{th,i}}^2/\nu_{\mathrm{p}}^{\mathrm{i/e}}$ $\sim D_{||}^{\mathrm{e}}$ $\quad \chi_{||}^{\mathrm{i}} \approx v_{\mathrm{th,i}}^2/\left(\nu_{\mathrm{E}}^{\mathrm{i/i}} + \cancel{\nu_{\mathrm{E}}^{\mathrm{i/e}}}\right)$ $< \chi_{||}^{\mathrm{e}}$ $< \nu_{\mathrm{E}}^{\mathrm{i/i}}$

*Or in an un-magnetised plasma

# Classical picture of diffusion across magnetic field lines in a highly ionised plasma



- Step-size (magnetised plasma!)
  - Ions: $\rho_{Li} = \sqrt{2m_i T_i}/(ZeB)$
  - Electrons: $\rho_{Le} = \sqrt{2m_e T_e}/(eB)$

$D_\perp = \nu \rho_L^2$ increases with collisionality!

- Collision frequencies

| | Momentum | Energy |
|---|---|---|
| e-i [$\nu_p^{e/i}$] | $1$ | $\sim m_e/m_i$ |
| e-e [$\nu_p^{e/i}$] | $\sim 1$ | $\sim 1$ |
| i-i [$\nu_p^{e/i}$] | $\sim\sqrt{m_e/m_i}$ | $\sim\sqrt{m_e/m_i}$ |
| i-e [$\nu_p^{e/i}$] | $\sim m_e/m_i$ | $\sim m_e/m_i$ |

- Collisions among particles of same species contribute to heat, but not particle transport

Electrons: $D_\perp^e \approx \nu_p^{e/i} \rho_{L,e}^2$ $\quad \chi_\perp^e \approx \left(\nu_E^{e/e} + \cancel{\nu_E^{e/i}}\right) \rho_{L,e}^2$ $\quad < \nu_E^{e/e}$

Ions: $D_\perp^i \approx \nu_p^{i/e} \rho_{L,i}^2$ $\sim D_\perp^{e/i}$ $\quad \chi_\perp^i \approx \left(\nu_E^{i/i} + \cancel{\nu_E^{i/e}}\right) \rho_{L,i}^2$ $> \chi_\perp^e$ $\quad < \nu_E^{i/i}$

# Overview of classical heat transport




Compare parallel and perpendicular heat transport in fusion plasmas

# Overview of classical particle transport





- For ions and electrons in thermal equilibrium ($T_e = T_i$) the classical perpendicular diffusion coefficients are

$$D_{\perp,\mathrm{i}} \approx \nu_\mathrm{p}^{\mathrm{i/e}} \rho_{\mathrm{L,i}}^2 = \nu_\mathrm{p}^{\mathrm{i/e}} \frac{m_\mathrm{i} T}{(eB)^2} = \nu_\mathrm{p}^{e/i} \frac{m_\mathrm{e} T}{(eB)^2} = \nu_\mathrm{p}^{\mathrm{e/i}} \rho_{\mathrm{L,e}}^2 \approx D_{\perp,\mathrm{e}}$$

  - ➢ Cross-field diffusion is automatically ambipolar and there is no need for an electric field to maintain charge neutrality (also parallel!)

- $D_\perp \propto n/\left(B^2\sqrt{T}\right)$, thus increasing the magnetic field reduces the classical cross-field diffusion

- $D_{||}/D_\perp = \left(\Omega_\mathrm{e}/\nu_\mathrm{p}^{\mathrm{e/i}}\right)^2 \approx 10^{16}$ for typical fusion plasmas (similar for e and i)

  - ➢ Parallel diffusion is much faster than the perpendicular diffusion
  - While $D_\perp$ increases with density, $D_{||} \propto T^{5/2}/n$ is reduced at higher density

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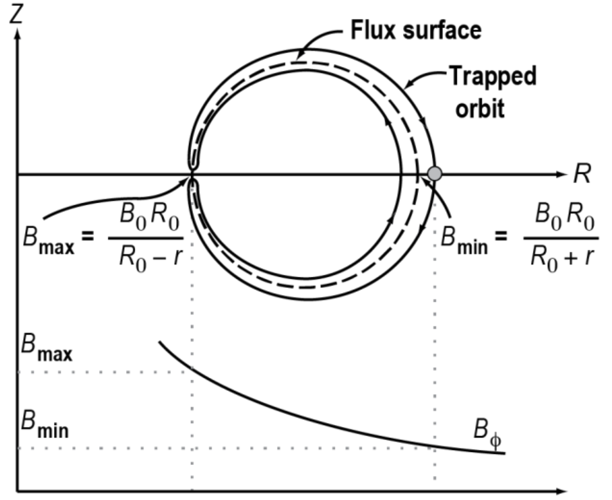
# Neoclassical diffusion in a tokamak

- Inhomogeneous magnetic field in a toroidal configurations results in drifts and magnetic mirrors (→ see E3-1)
  - Condition for trapping in the magnetic mirror

$$\frac{v_{||}^2}{v^2} < 1 - \frac{B_{min}}{B_{max}} = 1 - \frac{R_0 - r}{R_0 + r} \approx 2\frac{r}{R_0}$$

  - Condition applies to $\sqrt{2\varepsilon}$ of all particles
    ➔ trapped particle fraction




[J.P. Freidberg, PP and FE, Fig. 14.11]

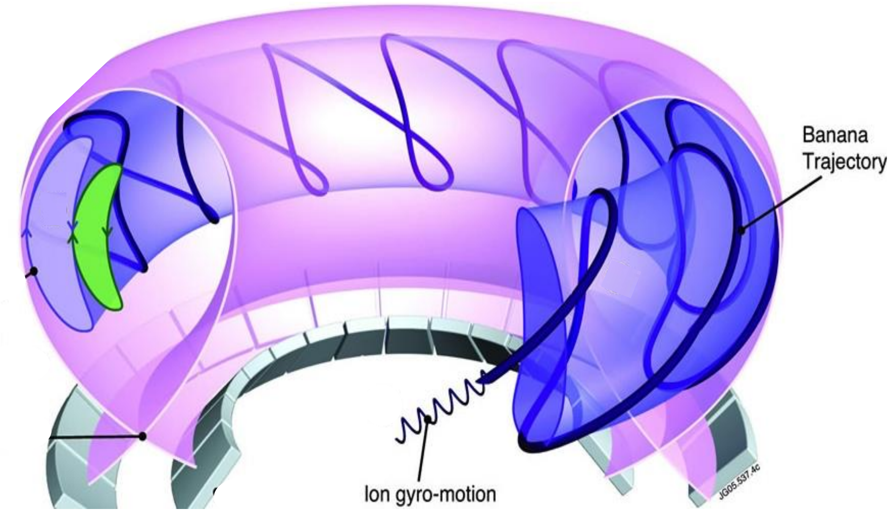
➢ Distinguish between trapped and passing particles

# Neoclassical diffusion in a tokamak





- Toroidal drifts ➔ trapped particles follow 'banana' orbits
  - Note that bananas also precess in the toroidal direction




➢ Drifts can affect transport, if particle can move through entire orbits before colliding

→ Orbit transit time $\tau_{\mathrm{TR}} \sim qR/v_{\parallel} < \tau_{\mathrm{coll}}$

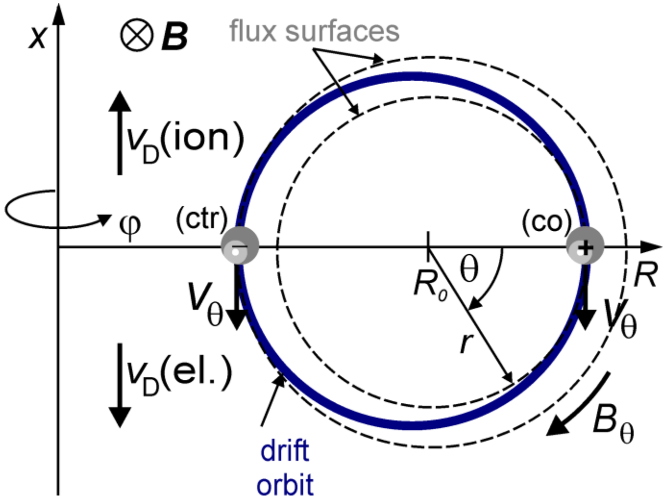
# Neoclassical diffusion in a tokamak: passing orbits





- Passing orbit follows flux surface continuously around the torus
  - Orbit displaced by $\delta_{\mathrm{DRIFT}} \approx q\rho_{\mathrm{L}}$
  - Effective collision frequency

$$\nu_{eff} = \nu/\varepsilon$$




- Diffusion coefficient

$$D_{PASS} = \left(1-\sqrt{2\varepsilon}\right)\nu_{eff}\,\delta_{\mathrm{DRIFT}}^2$$

$$= \left(1-\sqrt{2\varepsilon}\right)\frac{q^2}{\varepsilon}\,\rho_L^2\,\nu$$

Passing particle fraction

$D_{\perp,CLASS}$

  - Compare

$$D_{PASS} = \left(1-\sqrt{2\varepsilon}\right)\frac{q^2}{\varepsilon}D_{\perp,CLASS} \quad \Rightarrow \quad \begin{cases} r\to 0: & D_{PASS}(r) \approx \dfrac{q^2(r)}{\varepsilon(r)}D_{\perp,CLASS}(r) \\ r\to a: & D_{PASS}(r) \approx q^2(r)\,D_{\perp,CLASS}(r) \end{cases}$$

# Neoclassical diffusion in a tokamak: banana orbits

- Trapped banana orbit sits on LFS of flux surface
  - Orbit width: $\delta_{BAN} \approx q\rho_L/\sqrt{\varepsilon}$
  - Effective collision frequency:

$$\nu_{eff} = \nu/\varepsilon$$

- Diffusion coefficient

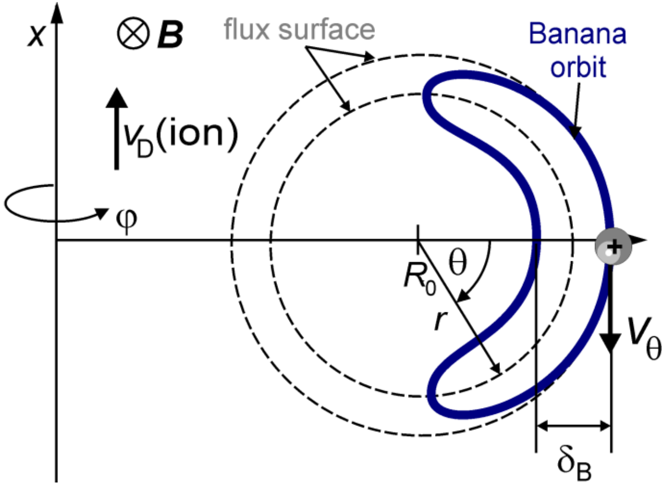
$$D_{BAN} = \sqrt{2\varepsilon}\ \nu_{eff}\,\delta_{BAN}^2 = \frac{\sqrt{2}q^2}{\varepsilon^{3/2}}\,\nu\,\rho_L^2$$

Trapped particle fraction

  - Compare

$$D_{BAN}(r) \approx \frac{\sqrt{2/\varepsilon(r)}}{1-\sqrt{2\varepsilon(r)}}\,D_{PASS}(r)$$

$$D_{BAN}(r) \approx \begin{cases} \sqrt{2/\varepsilon(r)}\,D_{PASS}(r) > D_{PASS}(r) \\ \dfrac{\sqrt{2}\,q^2(r)}{\varepsilon^{3/2}(r)}\,D_{\perp,CLASS}(r) \gg D_{\perp,CLASS}(r) \end{cases}$$



# Neoclassical diffusion in a tokamak: Pfirsch-Schlüter regime

- Pfirsch-Schlüter transport: short mean free path limit $\lambda_{\mathrm{MFP}} \ll Rq$
  - Even passing particles do not complete a full orbit
  - Parallel diffusion with $D_{||} = \nu_c \lambda_{\mathrm{MFP}}^2 = \frac{v_{th}^2}{\nu_c}$
- Time for a poloidal revolution

$$\tau_{pol} = \frac{(Rq)^2}{D_{||}} = \frac{(Rq)^2}{v_{th}^2}\nu_c$$

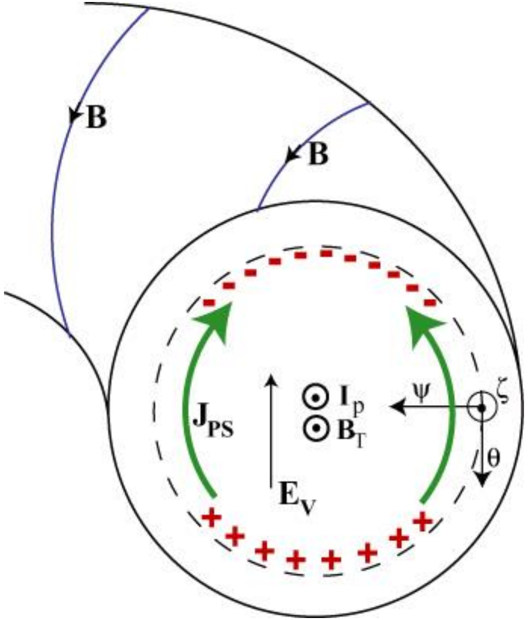
- Perpendicular drift (curvature and $\nabla B$)

$$v_{Drift} \sim v_{th}\frac{\rho_L}{R}$$

- Diffusion coefficient $D_{\mathrm{PS}}$

$$D_{\mathrm{PS}} = \frac{\left(v_{Drift}\tau_{pol}\right)^2}{\tau_{pol}} \sim q^2 \nu_c\, \rho_L^2$$

  - Compare $D_{PS} = q^2 D_{\perp,CLASS} \gg D_{\perp,CLASS}$

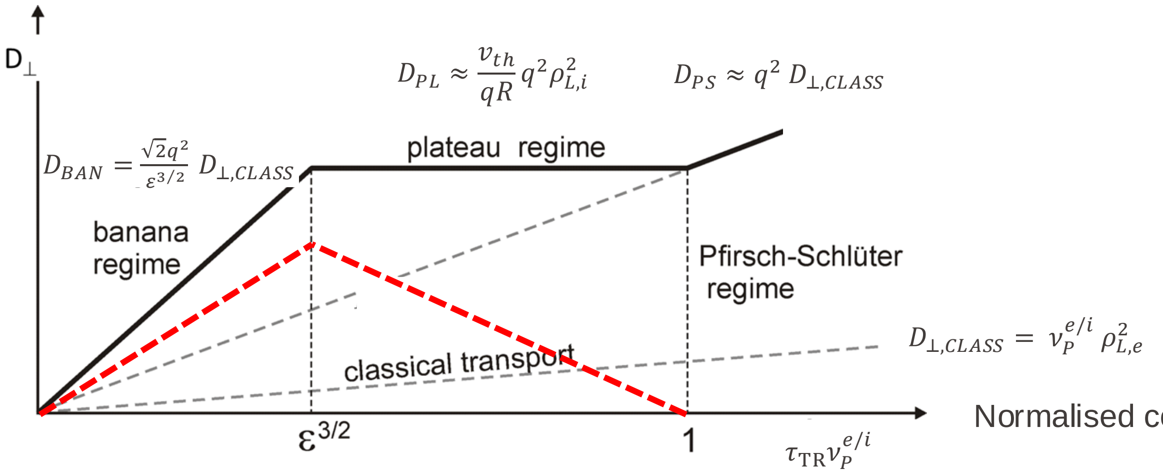


# Neoclassical diffusion in a tokamak: collisionality dependence



H. Reimerdes

- Scaling of the perpendicular diffusion coefficient $D_\perp$ as function of the collision frequency $\nu_P^{e/i}$ for momentum transfer between ions and electrons in different regimes of collisionality




Time between effective collisions > time to sufficient to bounce in the mirror

Number of collisions for each poloidal transit

# Neoclassical diffusion in a tokamak: typical numbers





- Typically $D_{\perp,\mathrm{NEO}} \approx (25-60) \times D_{\perp,\mathrm{CLASS}}$
- JET case (see **Exercise 2**)
  - $D_{\perp,\mathrm{CLASS}} \approx 5 \times 10^{-5}\, m^2/s$
  - $D_{\perp,\mathrm{NEO}} \approx (1-5) \times 10^{-3}\, m^2/s$
- However: measured diffusion typically $D_{\perp,\mathrm{meas}} \approx a^2/\tau_E \approx 1\, m^2/s$ → still much larger than $D_{\perp,\mathrm{NEO}}$!

➢ Must consider anomalous diffusion

# Classical vs. neo-classical diffusion in a tokamak



## Classical diffusion

- Tokamak approximated by a straight cylinder ➔ absence of magnetic field curvature
- Particle orbits = Larmor orbits
- Classical diffusion coefficient:

$$D_{\perp,\text{CLASS}} \sim \nu_{\mathrm{p}}^{\mathrm{e/i}} \rho_{\mathrm{L,e}}^{2}$$

## Neo-classical diffusion

- Tokamak as a toroidal device ➔ magnetic field curvature becomes important
- Particle orbits: more complex than simple Larmor orbits
- Neo-classical diffusion coefficient:

$$D_{\perp,\text{NEO}}(r) \sim q^2/\varepsilon^{\gamma} \times D_{\perp,\text{CLASS}}$$

  - $\gamma$ depends on relevant orbit, i.e. regime (Banana, plateau, Pfirsch-Schlüter)

**JET:**
- $D_{\perp,\text{CLASS}} \approx 5\times10^{-5}\,\text{m}^2/\text{s}$
- $D_{\perp,\text{NEO}} \approx (1\text{-}5)\times10^{-3}\,\text{m}^2/\text{s}$
- But $D_{\perp,\text{meas}} \approx 1\,\text{m}^2/\text{s}$ still $\gg D_{\perp,\text{NEO}}$ ➔ **Diffusion is anomalous!**

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1. General principle of diffusive transport: random walk
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5. **Turbulent transport in a tokamak ➔ anomalous diffusion**

# Turbulent fluctuations and diffusion

- **Electrostatic (ES) fluctuations**: $\delta E_\theta = -\nabla_\theta \phi$ produce an $\bar{E} \times \bar{B}$ drift velocity $\delta v_r = \delta E_\theta / B_0$
  - Lead to a convective flux

$$\delta\Gamma_{\mathrm{ES}} = \langle \delta v_r \delta n \rangle = \frac{1}{B_0} \langle \nabla_\theta \phi \delta n \rangle$$

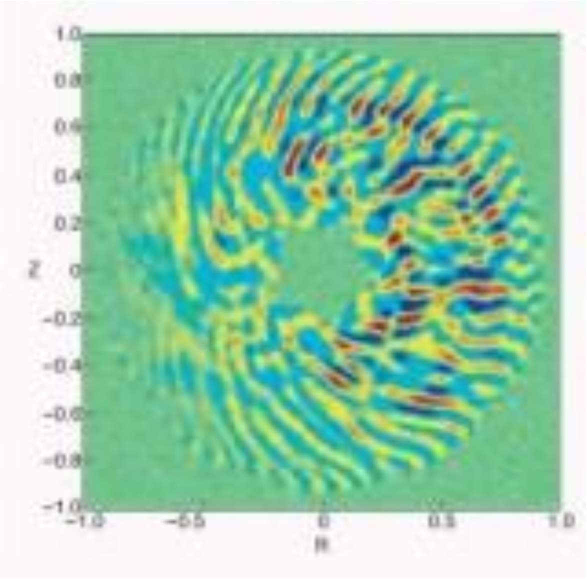
  - $\delta\Gamma_{\mathrm{ES}}$ vanishes, if the turbulent fields $\delta n$ and $\delta\phi$ are exactly in-phase
  - Various effects can introduce a phase shift and, hence finite $\delta\Gamma_{\mathrm{ES}}$
- **Electromagnetic (EM) fluctuations**: $\delta B_{\mathrm{r}}$ produce a convective particle flux

$$\delta\Gamma_{\mathrm{EM}} = \frac{n_0}{B_0} \langle \delta v_{||} \delta B_{\mathrm{r}} \rangle$$

- The instabilities causing ES and/or EM turbulent fluctuations typically occur over a scale length comparable to a few ion Larmor radii: micro-instabilities

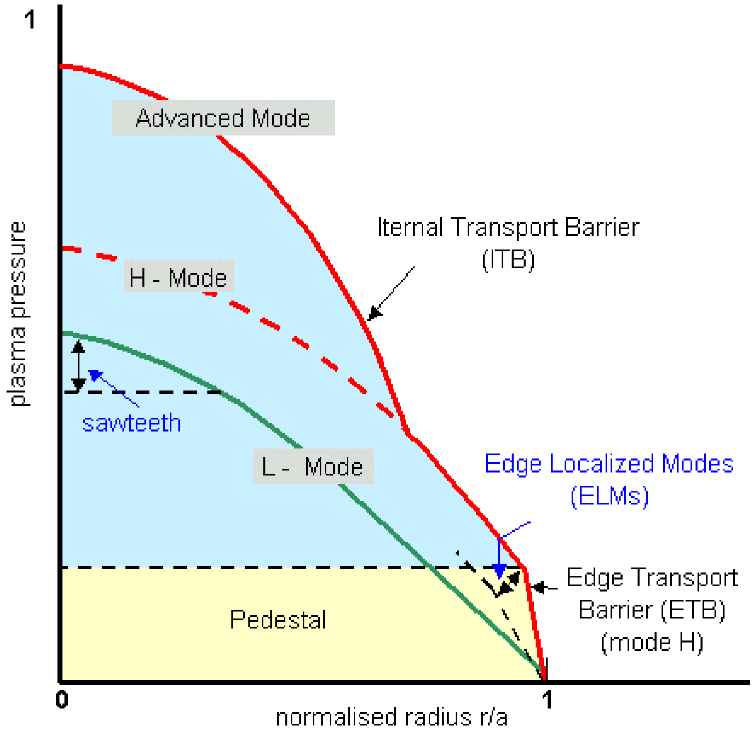
# Modelling anomalous diffusion

- Electric potential in a JET plasma turbulence simulation



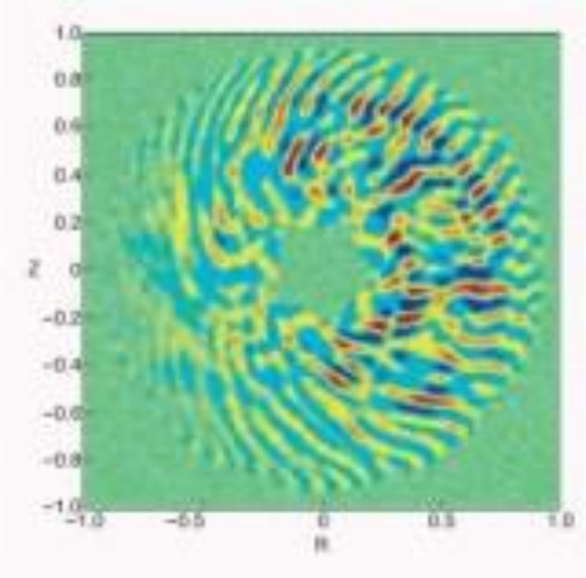
# Anomalous diffusion and transport barriers

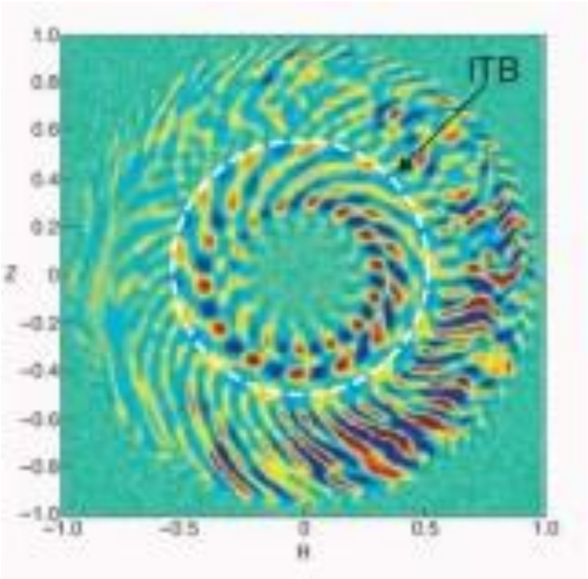
- **L-mode**: basic low-confinement regime ➔ $D_{\perp,\mathrm{meas}} \approx 500 \times D_{\perp,\mathrm{NEO}}$
- Reduce diffusion at plasma edge: **H-mode** ➔ edge transport barrier: $D_{\perp,\mathrm{meas}} \approx 5 \times D_{\perp,\mathrm{NEO}}$
- Reduce diffusion in plasma core: advanced scenario with internal transport barrier (**ITB regime**) ➔ $D_{\perp,\mathrm{meas}} \approx 2 \times D_{\perp,\mathrm{NEO}}$



# Modelling anomalous diffusion

- Electric potential in a JET plasma turbulence simulation
  - Reproduce plasma states without (left) and with (right) an Internal Transport Barrier (ITB)

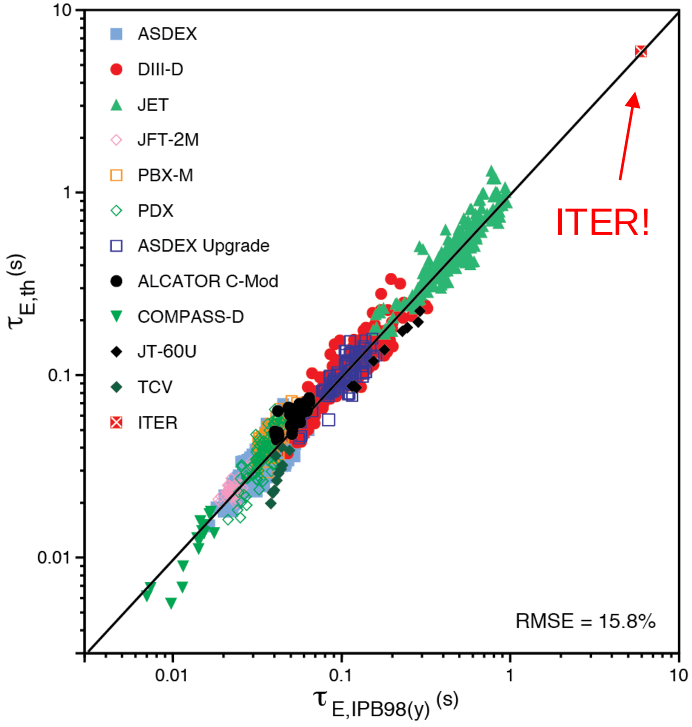




# The alternative: an empirical approach

- Pending a first principle theory of turbulent transport next steps such as ITER are planned based on empiric power scaling laws
  - Typically based on engineering parameters, e.g. IPB98(y)

$$\tau_{\mathrm{E,th}}^{\mathrm{ELMy}} = 0.0365\, I_{\mathrm{P}}^{0.97} B_{\mathrm{T}}^{0.08}\, P^{-0.63}\, n^{0.41} \times M^{0.20} R^{1.93} \varepsilon^{0.23} \kappa^{0.67}$$




*[E. Doyle, et al, Nucl. Fusion (1999)]*

# Turbulent transport: Summary




- Experimentally observed transport exceeds neo-classical transport by several orders of magnitude → anomalous transport
- Anomalous diffusion due to
  - Perturbations that allow transport across nested magnetic flux surfaces → electro-static turbulence
  - Perturbations that break-up nested magnetic surfaces allowing particles and energy to flow along newly formed field lines which follow stochastic trajectories → electro-magnetic turbulence
- Suppression of turbulent transport locally increases gradients → transport barriers
- Simulations of turbulent transport capable of reproducing observations